MODEL BENDING: TEACHING CIRCUIT MODELS NEW TRICKS

Daniel J. Gillespie and Samuel Schachter

Newfangled Audio
Fanwood, NJ
dan@newfangledaudio.com | schachtersam32@gmail.com

ABSTRACT

A technique is introduced for generating novel signal processing systems grounded in analog electronic circuits, called model bending. By applying the ideas behind circuit bending to models of nonlinear analog circuits it is possible to create novel nonlinear signal processors which mimic the behavior of analog electronics, but which are not possible to implement in the analog realm. The history of both circuit bending and circuit modeling is discussed, as well as a theoretical basis for how these approaches can complement each other. Potential pitfalls to the practical application of model bending are highlighted and suggested solutions to those problems are provided, with examples.

1. INTRODUCTION

Accurately modeling nonlinear analog circuits for audio production has been an active area of research for some time [1] [2] [3]. The most common approach is called white box modeling, which represents the circuit as a system of nonlinear differential equations [2] [4]. Building and solving these equations can be challenging because audio circuits often form transcendental nonlinear equations which require special techniques to solve [5]. However, the breakthroughs made in this field have greatly advanced our understanding of these nonlinear musical devices, preserved their sound, and made those sounds more widely available to musicians all over the world.

While it's important to preserve the sound of existing nonlinear audio equipment, it is also interesting to consider how our greater understanding of these systems can help us make novel musical sounds. While there are probably myriad ways in which our better understanding can aid in the development of new sounds, in this paper we will introduce a straightforward technique for creating novel musical sounds based on existing audio circuits which is analogous to the technique of circuit bending [6]. We call this technique model bending.

Circuit bending is the process of modifying an existing piece of audio electronics to create a new sound without the intent for a desired outcome, and without a theoretical basis for making the change [6]. It is like an experiment without a hypothesis; a physical exploration of the circuit that may lead to a chance discovery.

In a similar spirit, model bending is the process of building a model of a nonlinear analog circuit or system, then exploring the space of that model by making changes to it with the hope of finding interesting and novel sounds.

Section 2 of this paper will give a brief overview of circuit bending. Section 3 will give a brief overview of some common analog modeling techniques and highlight areas to consider when performing bends. Section 4 will introduce a more formal definition of model bending with some discussion. Section 5 will give three examples of model bending in practice and highlight how model bending is distinct from circuit bending. Finally, Section 6 will offer some conclusions and ideas for further work.

2. CIRCUIT BENDING

Circuit bending is a term coined by Reed Ghazala in 1992 [6] which refers to the process of treating an existing piece of audio electronics as the canvas for creating new sounds. This is done by disassembling the device and modifying the circuit with switches or other external components in ways that modify the audio output. These individual modifications are called bends and are shared within the circuit bending community both in person and online [7] [8] [9]. While the history of circuit bending can be traced to the 1960's or earlier [10], its popularity has exploded with the internet and circuit bent instruments have found their way into mainstream popular music over the last several decades [7] [11].

While the techniques for finding bends have expanded over the years, the technique described by Ghazala is simple [10].

1. Cut a 12" piece of insulated multi-strand wire, strip a little insulation off each end and "tin" the ends with solder to make them solid and firm.
2. With the circuit making a sound, touch one end of the wire to a circuit point and the other end of the wire to another circuit point.
3. If this results in an interesting sound, mark the circuit so you know where the ends of the wire were to create that new sound.
4. Keeping one end of the wire stationary and on the initial spot, the other end of the wire, let's call it the traveling end, is touched to another arbitrary spot.
5. If a new sound is created the circuit board is marked again.
6. Once the circuit is searched in this way, if the searcher is not yet content with the found sounds, all starts again but with the stationary end of the wire on a new spot. The traveling end repeats its tour.

This somewhat haphazard technique does not always generate useful results, but it does create a process of accidental discovery where useful new sounds can be, and are, found. This discovery process relies on the circuit bending principles of "anti-theory" and "immediate canvas."

Anti-theory refers to the idea of discovery without hypothesis; the idea that we will make a modification to the circuit without using prior knowledge to make a guess about the outcome. We simply modify and observe.

Meanwhile "immediate canvas" refers to the idea that there is no longer the need to start each instrument from scratch. Treating the existing instrument as the medium onto which a new work of art is created frees us to incorporate more ideas than only our own, and to do discovery more quickly. This transformation of existing art into a new work is similar to the disciplines of collage or sampling, where the artist uses existing work as the medium to create a new work.

Finally, Ghazala also writes about the "invention threshold," the idea that circuit bending only became possible once there was a proliferation of audio electronics available to be bent. Similarly, the proliferation of analog modeling research over the last couple decades has brought us to a place where it is inevitable to consider using circuit bending ideas to bend these models.

3. CIRCUIT MODELING TECHNIQUES

There are several common approaches to modeling nonlinear audio circuits. When implemented correctly these should all give the same results, but internally they represent the circuits quite differently, and these different representations may react differently to bending. The most common approaches are state space modeling, wave digital filters, and more recently, machine learning. We'll give a brief overview of each, with special attention paid to how they incorporate nonlinearities.

3.1. State Space Modeling

Electronic circuits are usually represented by the voltages across components and the currents flowing through them. State space modeling leverages this insight to build a system of nonlinear differential equations where the inputs, outputs, and states are the voltages across the components and the currents flowing through them.

While state space analysis is common in electrical engineering, the first attempt to create a comprehensive real-time technique for modeling audio circuits was the discrete K-method by Yeh [3]. The discrete K-method first discretizes all stateful elements in a circuit, primarily the capacitors and inductors, by creating companion circuits, then creates a new discrete nonlinear filter

$$\mathbf{x}[n] = \mathbf{A}\mathbf{x}[n-1] + \mathbf{B}\mathbf{u}[n] + \mathbf{C}\mathbf{i}[n] \quad (1a)$$

$$\mathbf{v}[n] = \mathbf{D}\mathbf{x}[n-1] + \mathbf{E}\mathbf{u}[n] + \mathbf{F}\mathbf{i}[n] \quad (1b)$$

$$\mathbf{y}[n] = \mathbf{L}\mathbf{x}[n-1] + \mathbf{O}\mathbf{u}[n] + \mathbf{Q}\mathbf{i}[n] \quad (1c)$$

$$\mathbf{i}[n] = f(\mathbf{v}[n]) \quad (1d)$$

where $\mathbf{u}[n]$ is the input vector, $\mathbf{y}[n]$ is the output vector, the states $\mathbf{x}[n]$ are defined as the voltages across capacitors and currents through the inductors at step n, and $\mathbf{v}[n]$ and $\mathbf{i}[n]$ are the voltages across the nonlinear elements and the currents induced by those voltages.

Matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}, \mathbf{L}, \mathbf{O}$, and $\mathbf{Q}$ represent the values of the circuit elements and the connections between them. Finally, $f(\cdot)$ is the nonlinear function representing the currents induced by the nonlinear elements in the circuit.

This efficient scheme works for the majority of nonlinear circuits, but a similar scheme by Holters and Zölzer expands this to all nonlinear circuits [12]. For this paper we will stick to the Yeh formulation which treats the nonlinearities as voltage controlled resistors.

3.2. State Space Model Nonlinearities

Discrete K models form a nonlinear ODE in which the states are represented as currents and voltages and which require the states and outputs to be solved as a nonlinear function of the current states and inputs. For common nonlinear circuit elements, this sets up a transcendental nonlinear equation which can be solved via iterative methods like Newton's method [5].

Stability in stateful nonlinear systems is a deep topic [13], but it is possible to guarantee that a circuit will remain stable if all elements in that circuit remain passive, because it will consume power. A nonlinear element will remain passive if it has both non-negative static and incremental resistance, as determined by it's $i-v$ characteristic curve.

If any element has a negative static resistance, $R_s = \frac{v}{i} < 0$, it produces power and is therefore not passive.

Additionally, nonlinear elements may have regions of negative incremental resistance, $R_i = \frac{\Delta v}{\Delta i} < 0$, where the slope of the $i-v$ curve is negative. These regions are also considered to be active, and may create multiple stable or unstable operating points [13].

Therefore, for a nonlinear element to remain passive its $i-v$ curve must stay in the first and third quadrants and the slope must never be negative. If these conditions are met for all elements in a circuit then the circuit will be guaranteed to remain stable for any input.

3.3. Wave Digital Filters

Wave digital filters (WDFs) are created from analog reference circuits by discretizing each electrical component in the circuit using an appropriate digital transform and mapping Kirchhoff variables (voltage v and current i) to wave variables (incident and reflected waves a and b). As originally introduced by Alfred Fettweis, wave digital filters were only able to model linear systems but more recent extensions to nonlinear systems [2] [14] [15] allow them to be used for all known nonlinear circuits.

The voltage wave definition defines the transformation between Kirchhoff and wave variables, and its inverse, as

$$a[n] = v[n] + i[n]R_p \qquad b[n] = v[n] - i[n]R_p \quad (2a)$$

$$v[n] = \frac{a[n] + b[n]}{2} \qquad i[n] = \frac{a[n] - b[n]}{2R_p} \quad (2b)$$

where arbitrary constant R_p is defined as the port impedance. Each circuit element is then defined as a *one-port* element, and parallel and series connections between one-ports are defined as *multi-port* adaptors.

When implementing a WDF structure, the one-port elements and multi-port adaptors become nodes in a tree graph. One node is chosen to be the *root* of the tree, then the adaptors and elements (the *leaves*) are connected according to their connections in the analog reference circuit. Each leaf must also be adapted, meaning that its port impedance R_p must be chosen such that the instantaneous dependency between its incident and reflected waves is eliminated, effectively removing all delay-free loops from the underlying filter structure. Once this tree is built, running the model is a matter of propagating the reflected waves from the leaves of the tree to the root, effectively integrating the information from the states, then propagating the incident wave back to the leaves, which updates the states [14].

Circuits with a single nonlinearity are relatively straightforward to integrate into the wave digital structure provided the nonlinear one-port element is chosen as the root node. This is due to the fact that the incremental impedance of a one-port nonlinearity is dependent on the incident wave entering it, so R_p would need to be continuously updated in order to adapt the element. Dynamic adaptation has been attempted [16] but is not the norm [1] [2].

3.4. Wave Digital Filter Nonlinearities

Nonlinear WDF models of real circuits also tend to create transcendental nonlinear equations, despite the fact that the WDF formulation is set up to remove the delay-free loops which cause the state to be a nonlinear function of itself. This is because most, if not all, electronic nonlinearities are modeled as either voltage controlled current sources, voltage controlled voltage sources, current controlled current sources, or current controlled voltage sources. For instance, when you transform the nonlinear voltage controlled current source

$$i = f(v) \tag{3}$$

into the wave domain using functions (2a), the result

$$b = a - 2R_p f\left(\frac{a+b}{2}\right) \tag{4}$$

is itself a transcendental nonlinear equation for the $f(\cdot)$ found in common circuit elements.

However, this is not a result of the wave digital filter formulation of the circuit; rather, it is a result of the nonlinearities created by real circuit elements, which operate in the Kirchhoff domain. It is mathematically possible to define a nonlinearity in which b is purely a function of a,

$$b = h(a), \tag{5}$$

which will enable the WDF model to have a closed form solution that does not require the use of resource-intensive iterative techniques to solve.

It can be shown that when a nonlinearity represents a voltage controlled resistor as in (3) that equation (5) can only exist in a closed form when the relation between i and v is invertible and $f(\cdot)$ and $h(\cdot)$ are differentiable [1]. For example, per [2], the invertibility of (3) is guaranteed for

$$\frac{di}{dv} > -\frac{1}{R_p}, \tag{6}$$

which by the analysis in Appendix 8 implies that

$$\frac{db}{da} > -1. \tag{7}$$

A similar result exists for the current controlled resistor $v = f(i)$ showing that the function is invertible as long as

$$\frac{db}{da} < 1. \tag{8}$$

Like state space models, if each electrical one-port element in the analog reference circuit is passive or lossless, then the resulting WDF is guaranteed to be *pseudopassive* upon discretization. The instantaneous *pseudopower* absorbed by a given port is defined as [17] [18]

$$p[n] = \frac{a^2[n] - b^2[n]}{R_p}. \tag{9}$$

If $p[n] \geq 0$ then the port is dissipating power and is thus pseudopassive. If a given WDF is composed entirely of pseudopassive elements, then pseudopower is conserved and the filter as a whole is guaranteed to be pseudopassive. Additionally, Fettweis shows that the pseudopassivity of a WDF is a sufficient and necessary condition for the filter's stability [17]. In other words, we can guarantee the power stability of the circuit model as long as at each port

$$|b[n]| \leq |a[n]|. \tag{10}$$

3.5. Machine Learning Models

A more recent line of investigation has been modeling nonlinear audio circuits with machine learning techniques.

One approach uses grey box modeling in which you build a nonlinear State Space model then linearize it using kernel regression [19] or neural networks [20]. Either of these implementations start with the nonlinear state space model of Section 3.1, however, instead of building an idealized model of the nonlinearity and solving it via an iterative equation, you connect measurement devices to a real circuit and learn a function describing the updates to the states $\mathbf{x}[n]$ and outputs $\mathbf{y}[n]$ based only on the previous input and state values. This removes the problem of needing to solve a transcendental nonlinearity and, in the case where you measure a real circuit, has the potential to better capture a real circuit in action, rather than an idealized model.

Others have used a full black box approach, modeling the system without regard for its structure. This has been done using deep neural networks [21], recurrent neural networks [22], and expert systems [23] [24].

4. MODEL BENDING

4.1. Model Bending Process

Similar to circuit bending, model bending is the process of modeling a nonlinear circuit to generate a nonlinear ODE, then treating the nonlinear ODE as a canvas for the discovery of new sounds. The process is straightforward.

1. Use an existing modeling technique to create a digital model of a nonlinear circuit - or start with an existing model.
2. Program a real-time implementation of that model and exercise it while listening to the output. (It can be useful to write long winded code here, so as to create more places for modification.)
3. Try to forget what you know about the model; treat it simply as some processing code.
4. Make a modification to that processing code, such as changing signs, constants or operators; we've had particular success in swapping out the nonlinear functions for other ones.
5. If the modification creates a desirable change in the sound of the model, make a note of it, this is a bend. Then return the processing code to its initial model-true state.
6. Continue through the processing code searching for other bends and making notes.
7. When you've found a number of bends you like you can either create new code paths which switch them on or off, or otherwise integrate them into your processing code as new parameters.

It is important to note that this process follows the circuit bending concept of anti-theory. The purpose of step 3 is to forget the goal of the original circuit and to just explore the space of the model, looking for something new and useful.

We also want to stress that this borrows from the circuit bending notion of using an “immediate canvas,” in this case the modeled circuit. You don’t even have to generate a new model, there are many published models which can be used as the canvas for new bends.

Many of the bends tried through this process will not create sounds worth pursuing, many will create no sound at all, or cause systemic or numerical instabilities. However, there is the opportunity to find new sounds worth pursuing using this technique, and when you do they will already exist inside a parameterized framework.

4.2. Model Bending vs Circuit Bending

If we model a circuit and bend it, is that the same as bending a circuit then modeling it?

No.

It is important to remember that the model is not the circuit. While it is possible to bend a model in exactly the same way you can bend a circuit, the transformations available to the model are much broader than the transformations available to the circuit.

For instance, analog audio circuits are only able to select from a relatively small number of nonlinear elements, such as diodes, transistors, vacuum tubes, or saturating op-amps (though there are others). This list excludes simple nonlinearities such as $y = \sin(x)$ or $y = \sqrt{|x|}$ which are impossible or impractical to approximate with analog circuit elements. For this reason, bending a mathematical model opens up an almost infinite space of new nonlinear possibilities which aren’t available in the analog realm.

Additionally, different modeling techniques represent the same circuit in different ways. While it’s *possible* to bend different models to have the same output, applying the bending process to different models will create different avenues of exploration which themselves result in different bends, as we will show in Example 5.1. Specifically, if we choose a nonlinear function of the wave variables, rather than of the Kirchhoff variables, then we can select a closed form nonlinearity which does not require an iterative solution.

5. EXAMPLES

5.1. Discrete K and WDF Diode Clippers

To demonstrate that bending a model is not the same as bending a circuit we will use both the discrete K-method and WDFs to model and bend the two-capacitor diode clipper shown in Figure 1.

The two-capacitor diode clipper is a nonlinear filter with high-pass and lowpass characteristics and a saturating nonlinearity. Capacitors C_H and C_L set the high-pass and low-pass cutoff frequencies respectively. The diodes’ $i - v$ characteristics are modeled using the Shockley large-signal model [25]

$$i(v) = I_s \left(e^{\frac{v}{\eta V_t}} - 1 \right), \tag{11}$$

where I_s is the diode’s saturation current, V_t is the thermal voltage, and η is the diode ideality factor.

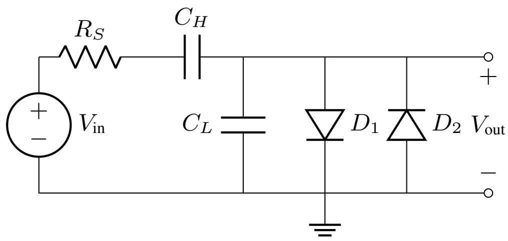

Figure 1: *Two-capacitor diode clipper circuit.*

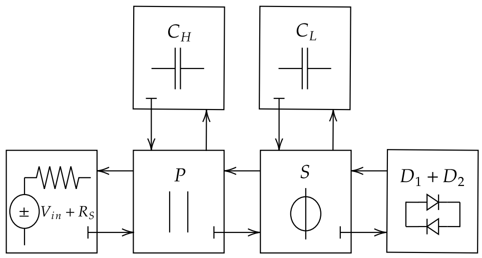

Figure 2: *Two-capacitor diode clipper WDF tree.*

5.1.1. Discrete K Diode Clipper Model Creation

The matrices for the discrete K-method implementation of the diode clipper were derived as shown in [3]. The state variables are set to be the voltages across the capacitors $\mathbf{x} = [V_{\text{CL}} V_{\text{CH}}]$ and the output voltage, V_{out}, is the voltage across capacitor C_L. The resulting matrices are

$$\mathbf{A} = \begin{bmatrix} -1/R_s C_L & -1/R_s C_L \\ -1/R_s C_H & -1/R_s C_H \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1/R_s C_L \\ 1/R_s C_H \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} -1/C_L & 0 \end{bmatrix}^T \quad \mathbf{D} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 0 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 0 \end{bmatrix}$$

and output matrices $\mathbf{L}$, $\mathbf{O}$, and $\mathbf{Q}$ are equal to the state matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ respectively since $V_{\text{out}} = V_{\text{CL}}$. These matrices can subsequently be plugged into (1) to solve the system of equations in real time.

The only unknown in the system of equations outlined in (1) is the nonlinear current $\mathbf{i}[n]$, which can be solved implicitly using Newton’s method as described in 3.2.

5.1.2. WDF Diode Clipper Model Creation

The WDF structure of the two-capacitor diode clipper is shown in Figure 2. The two anti-parallel diodes are combined into a single equivalent nonlinearity, which is set as the root of the connection tree. The voltage source V_{in} is set as the input, and the output is the junction voltage from the parallel adaptor. [3].

It is then necessary to derive the relationship between the incident and reflected waves of the diode pair one-port. We can use the definition of the wave relationship of the diode pair one-port as

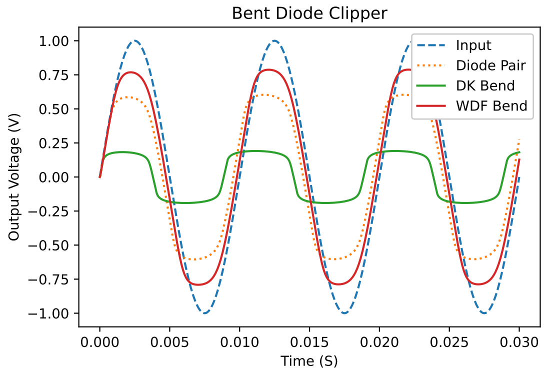

Figure 3: *(blue) Sinusoidal input with frequency 100Hz, amplitude 1V, (orange) output of NK model using (11), (red) output of WDF model using (14) and (15), (green) output of NK model using (13) and (15).*

defined in [25]

$$b(a) = \text{sgn}(a)\Big(|a| + 2R_pI_s - 2\eta V_t\mathcal{W}\big(\frac{R_pI_s}{\eta V_t}e^{\frac{R_pI_s+|a|}{\eta V_t}}\big)\Big) \quad (12)$$

where $\text{sgn}(\cdot)$ is the signum function.

5.1.3. Discrete K and WDF Bend Comparison

To show that we're bending the model rather than the circuit we will apply the same bend process to each of the discrete K-method and WDF models and show that they do not result in the same output, and therefore that the representation of the model indeed matters.

In an effort to generate new nonlinearities that are on the same scale as those created by a real diode, we will aim to keep the unit-bearing constants and form of the existing nonlinear functions where needed, and to replace only the nonlinear portion. This gives us the generalized nonlinear functions

$$i(v) = I_s\left(\tilde{f}\left(\frac{v}{\eta V_t}\right) - 1\right) \quad (13)$$

for the discrete K model and

$$b(a) = sgn(a)(|a| + 2R_pI_s - 2\eta V_t\tilde{f}(a)). \quad (14)$$

for the WDF model, where $\tilde{f}(\cdot)$ is the new nonlinearity, which for this example will be defined as

$$\tilde{f}(z) = 5z^8. \quad (15)$$

Figure 3 plots the output of the WDF and discrete K diode clipper models using a sinusoidal input signal of 100Hz, 1V amplitude. The chosen design values are $R_s = 200\Omega$, $C_L = 1\mu F$, $C_H = 5.3\mu F$, $V_t = 45.3mV$, $I_s = 1nA$, and $\eta = 1$.

The models produce identical outputs when using their prescribed nonlinear equations. However, when they are bent using (15), the models produce different output signals. While it should be possible to generate bends which create the same output using both the discrete K-method and WDFs, we show that starting with either a discrete K or WDF model, and applying the model bending process from Section 4.1 will result in very different outputs.

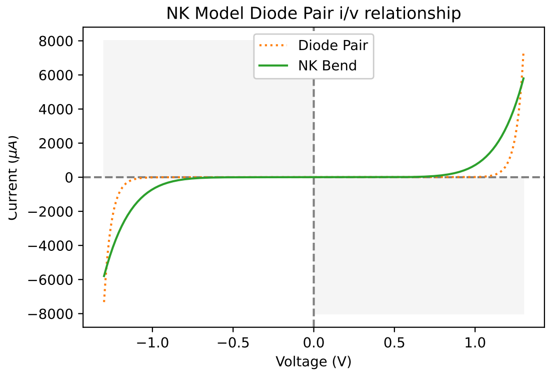

Figure 4: *(orange) Explicit diode pair nonlinearity, (green) nonlinearity produced by (13) and (15). The white area represents the passive region while the grey area represents the active region.*

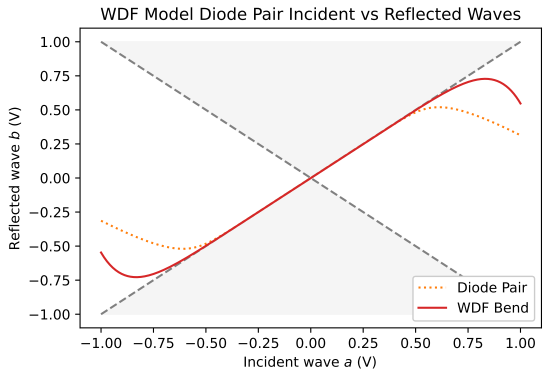

Figure 5: *(orange) Explicit diode pair nonlinearity, (red) nonlinearity produced by (14) and (15). The white area represents the region of pseudopassivity, the grey area represents the region of pseudoactivity, and the dashed lines represent pseudolosslessness [26].*

Additionally, Figures 4 and 5 demonstrate how the polynomial bend in (15), when applied to each model, produces a different nonlinearity to the diode pair. Figure 4 shows the diode's and bend's $i - v$ curves, while Figure 5 shows the diode's and bend's incident wave vs. reflected wave curves.

5.1.4. Other Diode Clipper Bends

When we bend the diode clipper circuit we are not limited to creating only clippers. As long as we follow the stability criteria outlined in Sections 3.2 or 3.4 we can implement novel nonlinearities that will still create complex interaction with the reactive elements in the circuit. Figure 6 shows the results of the following bend,

$$b = a\left(\frac{s - s\cos(|sa|) - R_p}{s - s\cos(|sa|) + R_p}\right) \quad (16)$$

where $s = 6.25$.

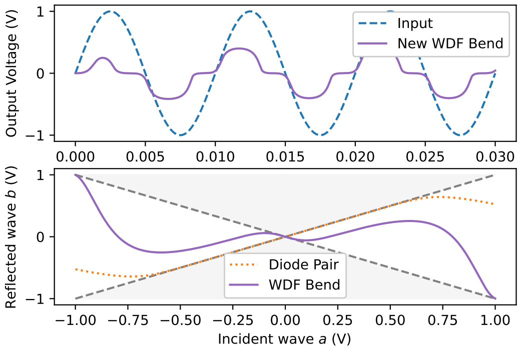

Figure 6: *Top: (blue) Sinusoidal input with frequency 100Hz, amplitude 1V, (purple) output of (16). Bottom: (purple) a-b characteristic of (16), (orange) diode a-b characteristic for reference.*

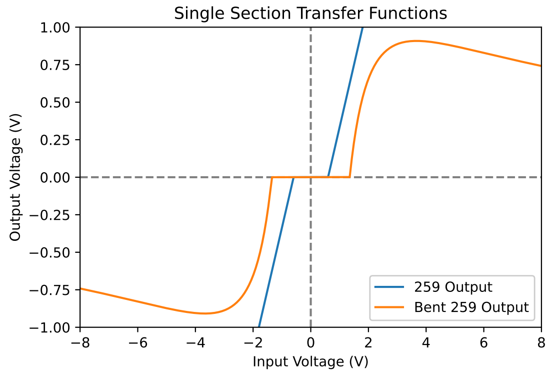

Figure 7: *Center clipping wavefolder sections from both the Buchla 259 (blue), produced by (18), and the Bent 259 (orange), produced by (23).*

5.2. Anti-Aliasing

When discretizing nonlinear audio systems it is important to keep in mind the effects of aliasing. Nonlinear systems generate overtones which may fall above the Nyquist frequency, which, if they are not suppressed, will fold down into the audible spectrum causing non-harmonic overtones which sound unpleasant. The simplest way to avoid aliasing is to run the nonlinear system at a very high sampling rate, thereby moving the Nyquist frequency high enough that no overtones will be generated above it. Unfortunately, for highly nonlinear systems the required sample rate will be quite high, making this method inefficient.

There are, however, other methods to suppress the artifacts caused by aliasing. One example is antiderivative anti-aliasing [27] [28].

In short, antiderivative anti-aliasing works by calculating the continuous-time antiderivative of the nonlinearity, applying this function in discrete time, then using a discrete-time difference equation to approximate the function's derivative, thereby applying an approximation to the original function. However, because the time antiderivative of a function has a low-pass characteristic, overtones that would otherwise be generated by the nonlinearity are suppressed, and are not recovered by the difference equation.

For implementation this requires us to have both the nonlinearity and its antiderivative on hand. This is a significant constraint when applying a model bending approach because antiderivatives are often difficult to find and many elementary functions have non-elementary antiderivatives [29].

Instead we'll show that it's possible to apply the model bending technique in the antiderivative domain, then find the continuous-time derivative of this novel nonlinearity (a comparatively simple procedure), to use in the ill-conditioned case.

5.2.1. Deriving the Antialiased 259

As an example we'll derive a bent form of the Buchla 259 wavefolder investigated by Esqueda et al., [30]. Note that while the Buchla 259 wavefolder is a stateless nonlinear system these same ideas are also applicable to stateful nonlinear systems.

The transfer function of the Buchla 259 wavefolder is generated by the weighted sum of five center clipping nonlinearities as shown in Figure 7, each generated by a single folding cell as shown

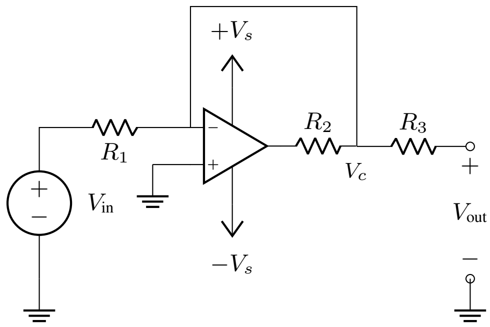

Figure 8: *Circuit for a folding cell in the Buchla 259*

in Figure 8, and each with its own R_1, R_2, and R_3 values. Despite the simplicity of each nonlinearity, the full circuit creates a highly nonlinear function as shown in Figure 9, which will create a large number of overtones, likely to alias.

We can define the transfer function of a single folding cell in terms of its slope s, offset o, and threshold t

$$s = \frac{R_3 R_2}{R_1 R_3 + R_2 R_3 + R_1 R_2} \tag{17a}$$

$$o = \frac{R_3 R_1 V_s}{R_1 R_3 + R_2 R_3 + R_1 R_2} \tag{17b}$$

$$t = \frac{R_1 V_s}{R_2} \tag{17c}$$

which are determined by the resistor values R_1, R_2, and R_3 given in the circuit and the rail voltage V_s.

The nonlinearity generated by a single folding cell can be written as

$$V_{\text{out}} = f(V_{\text{in}}) = \text{sgn}(V_{\text{in}})\max(s|V_{\text{in}}| - o, 0) \tag{18}$$

where V_{in} is the input voltage and V_o is the output voltage.

To apply antiderivative anti-aliasing to this circuit we must calculate the time antiderivative of V_{out} to find F_0

$$F_0(V_{\text{in}}) = \frac{sV_c^2}{2} - \text{sgn}(V_{\text{in}})V_c + c \tag{19}$$

which is itself in terms of the center clipped input voltage V_c and

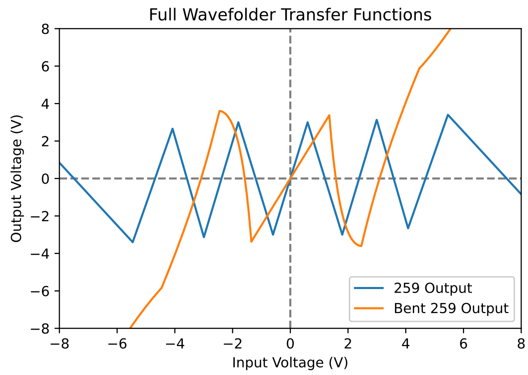

Figure 9: *(blue) Transfer function of the entire Buchla 259 clipper. (orange) Transfer function of the entire Bent 259 clipper.*

the constant of integration c

$$V_c = \text{sgn}(V_{\text{in}})\max(|V_{\text{in}}|, t) \tag{20a}$$

$$c = \frac{-R_2R_3t^2 + 2R_1R_3V_st}{2(R_1R_3 + R_2R_3 + R_1R_2)} \tag{20b}$$

Now that we have expressions for both $f(\cdot)$ and $F_0(\cdot)$ we can compute the antialiased output V_o^a [27]

$$V_o^a = \begin{cases} f\left(\frac{V_{\text{in}}[n]-V_{\text{in}}[n-1]}{2}\right) & V_{\text{in}}[n] \approx V_{\text{in}}[n-1] \\ \frac{F_0(V_{\text{in}}[n])-F_0(V_{\text{in}}[n-1])}{V_{\text{in}}[n]-V_{\text{in}}[n-1]} & otherwise. \end{cases} \tag{21}$$

5.2.2. Bending the Antialiased 259

In order to bend this antialiased implementation we will follow the steps in Section 4 to search through a number of perturbations of F_0. We've done this and selected one we'll call $\tilde{F_0}$

$$\tilde{F_0} = \frac{s\tilde{V_c}^2}{2} - \text{sgn}(V_{\text{in}})\tilde{V_c}o + c \tag{22a}$$

$$\tilde{V_c} = \text{sgn}(V_{\text{in}})\max(\ln(V_{\text{in}}^2), t). \tag{22b}$$

From this we can use a straightforward derivative to find the original function $\tilde{f}(V_i)$

$$\tilde{f}(V_i) = \begin{cases} \frac{2(s\tilde{V_c}-o)}{V_{\text{in}}} & \tilde{V_c} > t \\ 0 & \tilde{V_c} \leq t \end{cases}. \tag{23}$$

Which allows us to create an antialiased bent folding cell with the very different shape shown in Figure 7. Integrating five of these stages creates the bent 259 transfer function shown in Figure 9. Audio examples of all bends can be found at `http://newfangledaudio.com/modelbendingaudioexamples`.

6. CONCLUSIONS

In this paper we've introduced model bending, which formalizes a novel framework for using existing circuit models to explore and find new sounds. We've shown examples of applying this framework to discrete K-method and WDF models and addressed issues of resulting system computation, stability, and anti-aliasing. However, there is still a lot of area to explore.

In this paper we've focused primarily on replacing the nonlinearity in these nonlinear systems. This seems like it might be the most fertile ground to explore because true analog systems are limited in the types of nonlinearities they can practically produce, however, there is no reason why bends in other parts of the system wouldn't give interesting results.

We've also focused on bending white box models, but there might also be interesting insight to be gained by bending the grey or black box models which use machine learning. This has interesting potential since many machine learning techniques have geometric interpretations and bends could therefore be seen as geometric transformations on the space of signals.

Finally, this paper has been about bending circuit models, however, these same nonlinear ODEs are created when modeling acoustic and physical systems and all these techniques should be immediately applicable. We've had some success with our initial experiments, so this could be a promising line of inquiry.

7. REFERENCES

[1] K. Meerkotter and R. Scholz, "Digital simulation of nonlinear circuits by wave digital filter principles," in *IEEE Intl. Symposium on Circuits and Systems*, Portland, Or., May 8–11 1989, pp. 720–723.

[2] A. Sarti and G. De Poli, "Toward nonlinear wave digital filters," *IEEE Trans. on Signal Processing*, vol. 47, no. 6, pp. 1654–1668, June 1999.

[3] D. Yeh and J.O. Smith, "Simulating guitar distortion circuits using wave digital and nonlinear state-space formulations," in *Proc. Digital Audio Effects (DAFx-08)*, Espoo, Finland, Sept. 1–4 2008.

[4] D. Yeh, J. Abel, and J.O. Smith, "Automated physical modeling of nonlinear audio circuits for real-time audio effects—Part I: Theoretical development," *IEEE Trans. on Audio, Speech, and Language Processing*, vol. 18, no. 4, pp. 728–737, 2009.

[5] B. Holmes and M. Van Walstijn, "Improving the robustness of the iterative solver in state-space modelling of guitar distortion circuitry," in *Proc. Digital Audio Effects (DAFx-15)*, Trondheim, Norway, Nov. 30–Dec. 3 2015, pp. 1–8.

[6] Q. R. Ghazala, "What is circuit-bending, anti-theory & clear-illogic," *Experimental Musical Instruments*, vol. 14, no. 4, 1999.

[7] K.J. Werner and M. Sanganeria, "Bit bending: An introduction," in *Proc. Digital Audio Effects (DAFx-13)*, Maynooth, Ireland, Sept. 2–5 2013, pp. 1–7.

[8] S. Kuznetsov and E. Paulos, "Rise of the expert amateur: Diy projects, communities, and cultures," in *Proc. of the 6th Nordic Conference on Human-Comput. Interaction: Extending Boundaries*, Reykjavik, Iceland, Oct. 16–20 2010, pp. 295–304.

[9] A. Fernandez and F. Iazzetta, "Circuit-bending and the DIY culture," Tech. Rep., Escole de Communivações e Artes, Universidade de São Paulo, 2011.

[10] Q.R. Ghazala, "The folk music of chance electronics: Circuit-bending the modern coconut," *Leonardo Music Journal*, vol. 14, pp. 97–104, Dec. 2004.

[11] D. Deahl, "Circuit bending: Hacking a furby in the name of music," Available at https://www.theverge.com/2018/9/14 /17844906/circuitbending-hacking-a-furby-in-the-name-of-music, Accessed March 2022.

[12] M. Holters and U. Zölzer, "A generalized method for the derivation of non-linear state-space models from circuit schematics," in *2015 23rd European Signal Processing Conf. (EUSIPCO)*, Côte d'Azur, France, 31 Aug.–4 Sept. 2015, pp. 1073–1077.

[13] F. Miranda-Villatoro, F. Forni, and R. Sepulchre, "Dissipativity analysis of negative resistance circuits," *Automatica*, vol. 136, pp. 110011, 2022.

[14] K.J. Werner, *Virtual Analog Modeling of Audio Circuitry using Wave Digital Filters*, Ph.D. thesis, Stanford University, 2016.

[15] G. De Sancti and A. Sarti, "Virtual analog modeling in the wave-digital domain," *IEEE Trans. Audio, Speech, and Language Processing*, vol. 18, no. 4, pp. 715–727, May 2010.

[16] A. Bernardini and A. Sarti, "Dynamic adaptation of instantaneous nonlinear bipoles in wave digital networks," in *2016 24th European Signal Processing Conf. (EUSIPCO)*, Budapest, Hungary, Aug. 28 – Sept. 2 2016, pp. 1038–1042.

[17] A. Fettweis, "Pseudo-passivity, sensitivity, and stability of wave digital filters," *IEEE Trans. Circuit Theory*, vol. 19, no. 6, pp. 668–673, Nov. 1972.

[18] S. Bilbao, *Wave and Scattering Methods for Numerical Simulation*, J. Wiley & Sons, New York, NY, USA, 2004.

[19] D. Gillespie and D. Ellis, "Modeling nonlinear circuits with linearized dynamical models via kernel regression," in *IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, New Paltz, NY, Oct. 20–23 2013, pp. 1–4.

[20] J. Parker, F. Esqueda, and A. Bergner, "Modelling of nonlinear state-space systems using a deep neural network," in *Proc. Digital Audio Effects (DAFx-19)*, Birmingham, UK, Sept. 2–6 2019, pp. 1–8.

[21] E.P. Damskägg, L. Juvela, and V Välimäki, "Real-time modeling of audio distortion circuits with deep learning," in *Proc. Intl. Sound and Music Computing Conf. (SMC-19)*, Malaga, Spain, May 28–31 2019, pp. 332—339.

[22] A. Wright, E.P. Damskägg, and V. Välimäki, "Real-time black-box modelling with recurrent neural networks," in *Proc. Digital Audio Effects (DAFx-19)*, Birmingham, UK, September 2–6 2019, pp. 1–8.

[23] F. Eichas and U. Zölzer, "Black-box modeling of distortion circuits with block-oriented models," in *Proc. Digital Audio Effects (DAFx-16)*, Brno, Czech Republic, September 5–9 2016, pp. 1–7.

[24] F. Eichas and U. Zölzer, "Gray-box modeling of guitar amplifiers," *Journal of the Audio Engineering Society*, vol. 66, no. 12, pp. 1006–1015, Dec. 2018.

[25] R.C.D. Paiva, S. D'Angelo, J. Pakarinen, and V. Välimäki, "Emulation of operational amplifiers and diodes in audio distortion circuits," *IEEE Trans. Circuits and Systems II: Express Briefs*, vol. 59, no. 10, pp. 688–692, Oct. 2012.

[26] K.J. Werner and J.O. Smith, "An energetic interpretation of nonlinear wave digital filter lookup table error," in *Intl. Symposium on Signals, Circuits and Systems (ISSCS)*, Iasi, Romania, July 9–10 2015, pp. 1–4.

[27] J. Parker, V. Zavalishin, and E. Le Bivic, "Reducing the aliasing of nonlinear waveshaping using continuous-time convolution," in *Proc. Intl. Conf. Digital Audio Effects (DAFx-16)*, Brno, Czech Republic, Sept. 5–9 2016, pp. 137–144.

[28] S. Bilbao, F. Esqueda, J. Parker, and V. Välimäki, "Antiderivative antialiasing for memoryless nonlinearities," *IEEE Signal Processing Letters*, vol. 24, no. 7, pp. 1049–1053, July 2017.

[29] B. Conrad, "Impossibility theorems for elementary integration," in *Academy Colloquium Series*, Cambridge, MA, 2005.

[30] F. Esqueda, H. Pöntynen, V. Välimäki, and J. Parker, "Virtual analog Buchla 259 wavefolder," in *Proc. Intl. Conf. Digital Audio Effects (DAFx-17)*, Edinburgh, UK, Sept. 5–9 2017, pp. 192–199.

8. APPENDIX: ANALYSIS ON INVERTIBILITY OF WDF NONLINEARITIES

8.1. Voltage Controlled Resistor

A voltage controlled resistor of the form (3) may only be represented in the wave-domain form (4) if condition (6) is valid [2]. We can differentiate equations (2) with respect to voltage to find (we temporarily discard the step "[n]" as it is not pertinent to this derivation)

$$\frac{da}{dv} = 1 + \frac{di}{dv}R_p \quad (24) \qquad \frac{db}{dv} = 1 - \frac{di}{dv}R_p. \quad (25)$$

By application of the chain rule we find

$$\frac{db}{da} = \frac{1 - R_p\frac{di}{dv}}{1 + R_p\frac{di}{dv}}. \quad (26)$$

By applying equation (6) to equation (26) we can show that that $b = h(a)$ represents an invertible voltage controlled resistor *iff*

$$h'(a) > -1. \quad (27)$$

8.2. Current Controlled Resistor

Similarly, a current controlled resistor described by

$$v = f(i) \quad (28)$$

will only be valid when [2]

$$\frac{dv}{di} > -R_p. \quad (29)$$

Differentiating equations (2) with respect to current instead of voltage and applying the chain rule yields

$$\frac{db}{da} = \frac{\frac{dv}{di} + R_p}{\frac{dv}{di} - R_p}. \quad (30)$$

By applying equation (29) to equation (30) we can show that $b = h(a)$ represents an invertible current controlled resistor *iff*

$$h'(a) < 1. \quad (31)$$