BISTABLE DIGITAL AUDIO EFFECT

Alexander Ramirez, Vikas Tokala, Antonin Novak, Frederic Ablitzer and Manuel Melon

Laboratoire d'Acoustique de l'Université du Mans
LAUM - UMR 6613 CNRS, Le Mans Université,
Avenue Olivier Messiaen, 72085 LE MANS CEDEX 9, France
antonin.novak@univ-lemans.fr

ABSTRACT

A mechanical system is said to be bistable when its moving parts can rest at two equilibrium positions. The aim of this work is to model the vibration behaviour of a bistable system and use it to create a sound effect, taking advantage of the nonlinearities that characterize such systems. The velocity signal of the bistable system excited by an audio signal is the output of the digital effect. The latter is coded in C++ language and compiled into VST3 format that can be run as an audio plugin within most of the commercial digital audio workstation software in the market and as a standalone application. A Web Audio API demonstration is also available online as a support material.

1. INTRODUCTION

When the first audio effects were introduced into the market, musicians, composers, filmmakers, and audio engineers started experimenting with these gadgets obtaining new exotic and fascinating sounds that would revolutionize music forever. Since then, many enthusiasts have dedicated extensive hours to creating and synthesizing new sounds that can be used for musical compositions and for audiovisual productions. With the rapid development of the computer industry and digitalization of analogue audio effects [1], it has become even easier to experiment with them.

Many of these effects are using some nonlinear relation, which is of no surprise. Indeed, many beautiful sounds created by musical instruments rely very often on a nonlinear mechanism. Self-sustained oscillations in instruments like violins, flutes or trumpets, depend upon a severe nonlinearity [2, 3]. Even instruments based on free oscillations can exhibit nonlinear behaviour, for instance, due to vibrations of strings at large amplitude [4] or against unilateral contact obstacle [5]. Percussion instruments like cymbals or gongs may even exhibit chaotic behaviour [6, 7], an extreme form of nonlinear behaviour.

Many of the nonlinear digital audio effects are based on purely mathematical models going from simple static or dynamic nonlinear functions [1] to more complicated block-oriented models [8, 9]. Other nonlinear effects are based on physics [10, 11] or on modern techniques such as deep learning [12] to imitate the physics.

This work aims to study the main properties of bistable systems, that are described in section 2, and implement them into a real-time digital audio effect (section 3) capable of generating unique sounds driven by the nonlinear features of the system. Bistability, a phenomenon that arises in many real-world systems, means that a dynamical system can have two stable equilibrium states [13]. Such a system can lead to many kinds of nonlinear behaviours [14] and consequently to many interesting effects that can be implemented in digital domain.

2. BISTABLE SYSTEMS THEORY

This section describes the theory of bistable systems. It starts with a brief introduction to monostable linear and nonlinear systems with transitions into the bistable nonlinear systems and their properties. A mechanical prototype was build and video-recorded as a pedagogical support for the theory. The bistable system is next written in form of difference equations.

2.1. Bistable system properties

To understand what a bistable system is and how it works, one can start with a simple one degree of freedom (1DOF) linear mass-spring system (Fig. 1) with well known equations. It consists of a mass m connected to the ground by a linear spring of stiffness value k and a damper of value c. It is excited by an external force f, and x is the displacement of the mass. The equation of motion of such a system is [15]

$$m\ddot{x} + c\dot{x} + kx = f(t), \tag{1}$$

where $x \equiv x(t)$ is a function of time and where $\dot{x}$ and $\ddot{x}$ represent the first and second time derivative, respectively. The resonance frequency of such a system is expressed as

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{2}$$

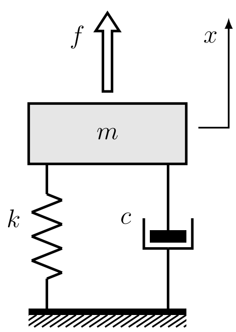

Figure 1: *Damped mass-spring system.*

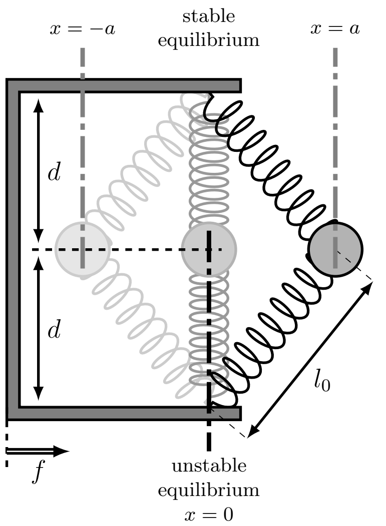

Figure 2: *Bistable system.*

By introducing a nonlinear stiffness, the monostable linear system can become nonlinear [16]. The equation that describes such a system, called the Duffing equation [17], is

$$m\ddot{x} + c\dot{x} + \alpha x + \beta x^3 = f(t), \tag{3}$$

where β is the nonlinear stiffness parameter. Such a system can also be used to generate audio effects [18, 19].

A bistable system can be obtained by adjusting the values of α and β in Eq. (3), as explained below. A 1DOF scheme of a bistable system is shown in Fig. 2. The system is presented as a mass attached by two fixed springs, whose free length l_0 (length of the spring when it is not under tension) is greater than the half-width of the frame [13], *i.e.* $l_0 > d$, a necessary condition for this system to be bistable. This structure exhibits two equidistant stable positions and an unstable region in between them. If $l_0 \leq d$ the system is monostable. The excitation force f in this system is applied from the base, which generates a mass displacement as a consequence.

The governing equation of motion of a bistable system is

$$m\ddot{x} + c\dot{x} - k_1 x + k_3 x^3 = f(t), \tag{4}$$

where $f(t)$ is the excitation force, x defines the displacement of the mass, m is the mass, c is the damping factor, and k_1 and k_3 determine the nonlinear stiffness. Note, that a bistable system is a special type of the more general Duffing System with a negative linear stiffness coefficient.

Essentially, the bistable system behaves as a monostable system with a nonlinear stiffness (stiffness depending on displacement) expressed as a polynomial in the terms $-k_1 x + k_3 x^3$, shown in Fig. 3. The curve obtained represents the relationship between the force f and mass displacement x of the system.

The elastic potential energy for the nonlinear stiffness, determined by $\int F(x)dx$, where $F(x)$ are the conservative forces $F(x) = -k_1 x + k_3 x^3$, is

$$PE = -\frac{1}{2}\, k_1\, x^2 + \frac{1}{4}\, k_3\, x^4. \tag{5}$$

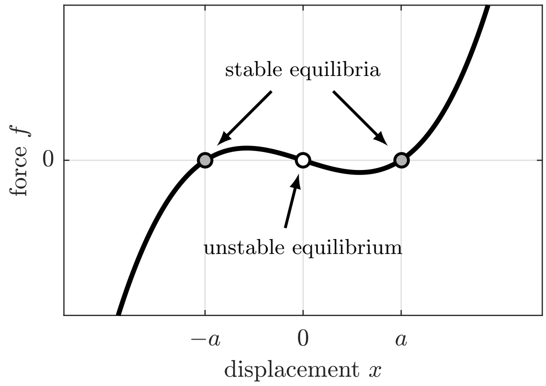

Figure 3: *Force Displacement Relationship of a bistable system.*

The lowest points on the energy potential curve shown in Fig. 4 indicate the two stable positions of the system. It also shows that the system is unstable in between these positions, and with a small perturbation, it is forced to move to one of the stable positions.

It can be noted that the potential energy is maximum at the unstable position, which prompts the system to move towards stable positions with minimal energy input. This is also a unique characteristic of the bistable system.

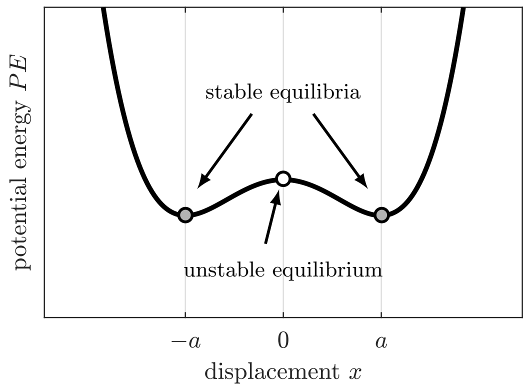

Figure 4: *Stored Potential Energy of a bistable system.*

2.2. Steady State Dynamics of Bistable Systems

One of the main characteristics of bistable systems is the presence of different steady-state dynamics resulting from the same excitation conditions. Particularly, this means that the dynamics of the system can exhibit different behaviour in two experiments, even if the excitation conditions are the same. The specific dynamic that occurs in the system is also dependent on the initial conditions.

Commonly, they can be grouped into two regimes, intrawell and interwell oscillations. The intrawell oscillations occur when the mass vibrates around one of the two stable positions. The interwell oscillations occur when the mass vibrates across the unstable position. Figs. 5 and 6 show the expected behaviour of the intrawell and interwell oscillations.

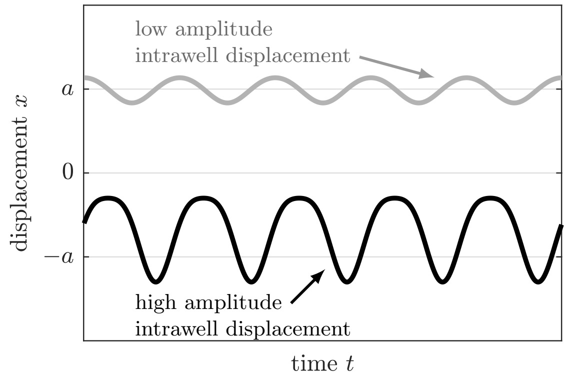

Figure 5: *Intrawell Oscillations of a bistable system.*

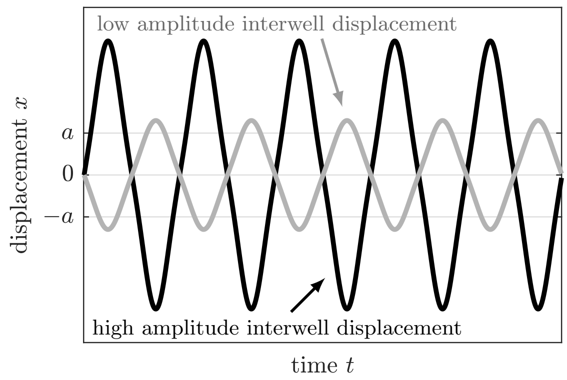

Figure 6: *Interwell Oscillations of a bistable system.*

A system dynamic where the system moves from one stable equilibrium to another crossing the unstable zero displacement position is called a *snapthrough*. Interwell oscillations are made of these snapthroughs.

Some nonlinear systems can oscillate aperiodically when subjected to sinusoidal excitations. In contrast with noise-induced behaviours, this characteristic is not random and can be determined with the equation that describes the bistable system. An example of this chaotic effect is shown in Fig. 7. In this figure, one can notice chaotic fluctuations in the displacement around the stable (intrawell oscillations) and unstable position (interwell oscillations).

2.3. Mechanical Demonstrator

To support the above-mentioned theory, a mechanical demonstrator was made and video recorded. The aim of this demonstrator is purely pedagogical: to demonstrate the dynamics of a bistable system by visualizing its motion through a real-world mechanical system.

The demonstrator is made of a post-buckled beam with central mass [20]. A rectangle cut out from a transparent plastic sheet (hereafter referred to as the membrane) was assembled with a metal washer, as shown in Fig. 8. To generate mechanical actuation, an old speaker was salvaged, and the cone and surround were removed. A 3D printed cylindrical cap is fitted with a clamping mechanism made from wood. A 3D printed spider is used to keep the actuating cylindrical cap in place and also to prevent the collision of the coil former to sides of the magnet. The membrane is fixed between two wooden clamps. When the loudspeaker is fed with a signal, it makes the former clamp-cap assembly move and, in turn, excites the bistable membrane.

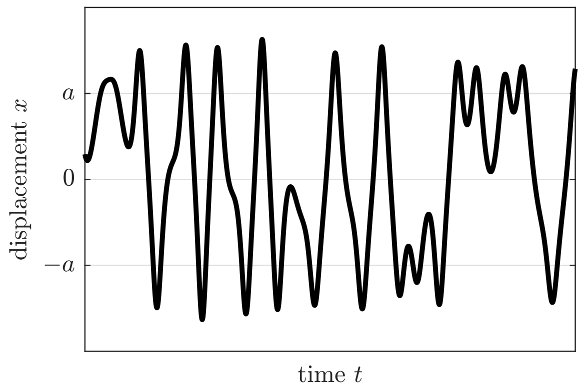

Figure 7: *Chaotic Oscillations of a bistable system.*

The entire assembly is mounted on a wooden platform for better stability. A video recording of this mechanical system demonstrating the intrawell and interwell oscillations is available online at `https://ant-novak.com/bistable_effect` [21].

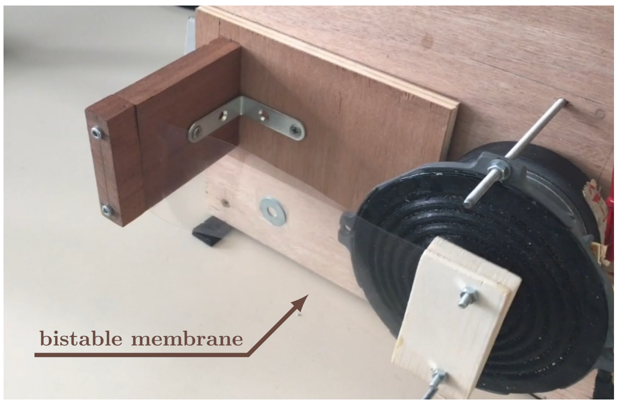

Figure 8: *A picture of the mechanical prototype to demonstrate the bistable system dynamics, video online at* `https://ant-novak.com/bistable_effect` *[21].*

2.4. Bistable System Discretisation

Since the objective of this work is to implement a bistable system in the digital domain we need to transform the governing differential equation of bistable system (Eq. (4)) to a difference equation. This is done using difference equation approximation [22] by setting

$$x \rightarrow x[n-1] \tag{6}$$

$$\dot{x} \rightarrow \frac{x[n]-x[n-1]}{T_s} \tag{7}$$

$$\ddot{x} \rightarrow \frac{x[n]-2x[n-1]+x[n-2]}{T_s^2} \tag{8}$$

$$f(t) \rightarrow f[n], \tag{9}$$

where T_s is the sampling period.

The resulting difference equation of bistable system is

$$x[n] = \frac{f[n-1] - A_2 x[n-1] - A_3 x[n-2] - k_3 x^3[n-1]}{A_1}, \tag{10}$$

where

$$A_1 = \frac{m}{T_s^2} + \frac{c}{T_s}, \tag{11}$$

$$A_2 = -\frac{2m}{T_s^2} - \frac{c}{T_s} - k_1, \tag{12}$$

$$A_3 = \frac{m}{T_s^2}, \tag{13}$$

and where m is the mass, c is the damping, and k_1 and k_3 are the linear and nonlinear stiffness, respectively. A_1, A_2 and A_3 are constants obtained from the finite differences method to solve the differential equation numerically.

Note, that all the physical quantities, such as mass m, damping c, or stiffness k, are, for the sake of simplicity, chosen unit-less. The digital input of the system $f[n]$ (corresponding to the force), as well as the digital outputs $x[n]$ and $\dot{x}[n]$ (corresponding to displacement and velocity), are also unit-less variables.

The initial value of the memory variables $x[n-1]$ and $x[n-2]$ determine the initial conditions of the system, according to

$$x[n-2] = x_0, \tag{14}$$

$$x[n-1] = \dot{x}_0 \cdot T_s + x_0, \tag{15}$$

where x_0 and $\dot{x}_0$ are the corresponding initial values for displacement and velocity. For the sake of simplicity, the initial conditions of the sound effect are set to zero.

Note also that the current implementation using difference equation approximation with a third-order polynomial necessarily inherits common problems related to discrete-time implementation of non-linear analog systems. The non-linear cubic operation can extend the bandwidth and cause non-linear aliasing. There are methods that try to avoid these problems [23, 24] but since they are beyond the scope of this paper, we keep the implementation simple and suggest using high sample rates or oversampling methods if necessary.

3. BISTABLE DIGITAL AUDIO EFFECT

This section presents the main results of our work. First, the difference equations of the bistable system are implemented in Matlab and are studied as a single-input, single-output nonlinear system. The excitation force $f[n]$ is used as the system input and the velocity $\dot{x}[n]$ is used as the output. Note that the displacement $x[n]$ (Eq. (10)) cannot be directly used as an output since the speakers or headphones that would be used to reproduce it are monostable systems. To give an example, for very small amplitudes, the bistable system stays around one of its stable positions, for example $x = +1$. Therefore, the membrane of the loudspeaker that is used to reproduce the output signal would be displaced from its rest position (if a DC-coupled amplifier was used). This would force the speaker to move like a bistable membrane, which could cause unwanted distortion and potential damage to the speaker. The loudspeaker membrane would be in an offset position even if no input signal was provided.

Next, the steady-state dynamics, and potential energy, are studied with respect to the final audio effect and the choice of range of parameters is explained. Finally, since the best way to evaluate the effect of bi-stability as a digital audio effect is to listen to it, we prepared two realizations of the digital audio effect: 1) a VST-plugin, 2) an online web audio applet. References to both are provided at the end of this section.

The C++ code of the main core of the implemented digital auido effect is provided in Appendix A.

3.1. Steady-state Dynamics

To excite the system described by Eq. (10) a harmonic signal $f[n] = F_0 \cos(\omega\, n\, T_s)$, with angular frequency $\omega = 1000$ rad/s is used. First, the *intrawell* oscillations are studied for $F_0 = 0.1$ and $F_0 = -0.6$. The sign of F_0 is chosen different to achieve positive or negative stable positions.[1] The damping is set to $c = 10^{-3}$, the linear and nonlinear stiffness coefficients are set to $k_1 = k_3 = 1$, and the mass is set as $m = k_1/\omega^2$. It can be seen in Fig. 9 that the system can exhibit the intrawell oscillations around one of the stable positions ($x = \pm 1$).

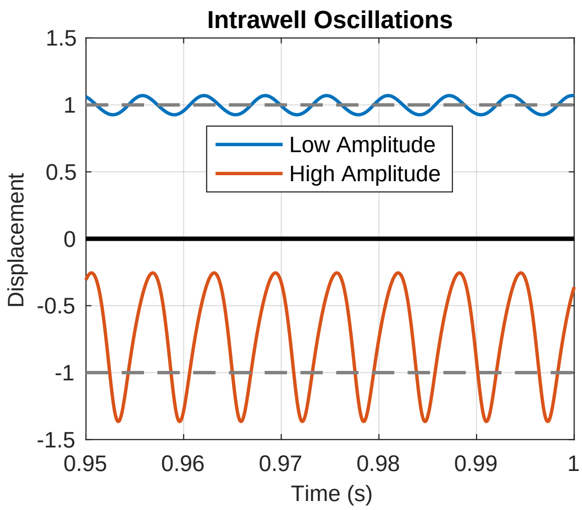

Figure 9: *Intrawell bistable system dynamics.*

In Figs. 10 and 11 the interwell oscillations of the bistable system are shown. In this particular example, the linear and nonlinear stiffness coefficients are set to $k_1 = k_3 = 1$, the damping coefficient $c = 10^{-4}$, and the mass is set as $m = k_1/\omega^2$. In Fig. 10 the periodic cases of the interwell oscillations are shown for low amplitude $F_0 = 0.3$ (blue curve) and for high amplitude $F_0 = -10$ (red curve). Setting the amplitude to $F_0 = 0.25$ leads to an aperiodic interwell behaviour as shown in Fig. 11.

[1]Note that the positive or negative stable position can be achieved by changing the initial conditions

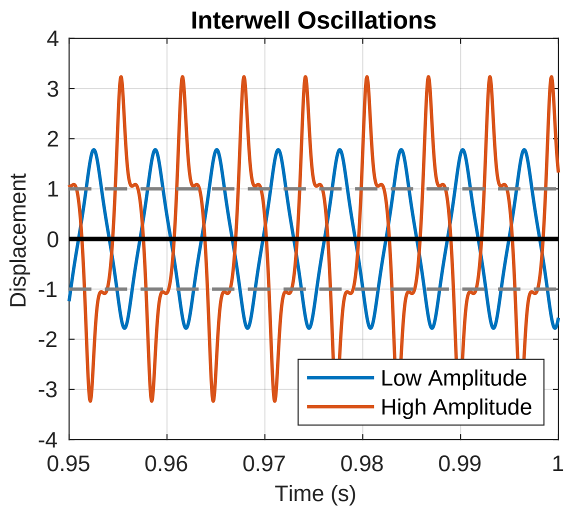

Figure 10: *Interwell bistable system dynamics.*

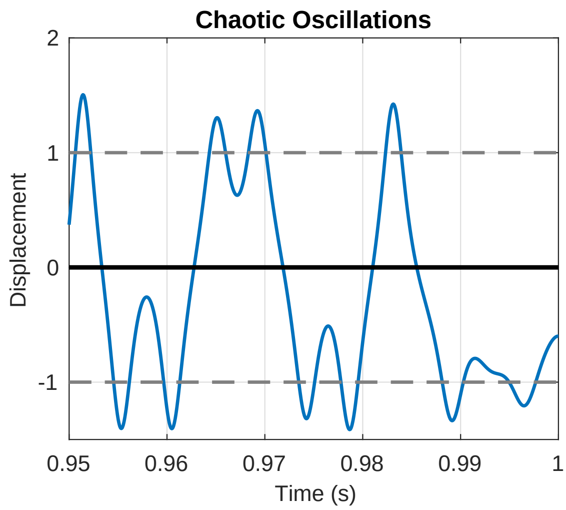

Figure 11: *Chaotic oscillations in the bistable system.*

The most important factors that determine whether the system oscillates with intrawell or interwell oscillations are k_1 and k_3. When the values for k_1 and k_3 are low, it is easier for the system to move between the stable positions as the stiffness is low and less force is needed to move the mass. On the contrary, when k_1 and k_3 are high, a greater force is required to move the mass between the two stable positions.

3.2. Potential Energy and Force-Displacement Relationship

The variation of the potential energy of a bistable system with displacement for different values of k_1 and k_3 is shown in Fig. 12. To simplify the study, k_1 is chosen equal to k_3, and also to establish the stable equilibrium positions of the system at $x = -1$ and $x = 1$ for all scenarios.

Another essential characteristic curve of the bistable system is the Force-Displacement curve, shown in Fig. 13 for several values of $k_1 = k_3$. It shows that as the stiffness of the system increases, a large amount of force is required to move the system out of its equilibrium positions.

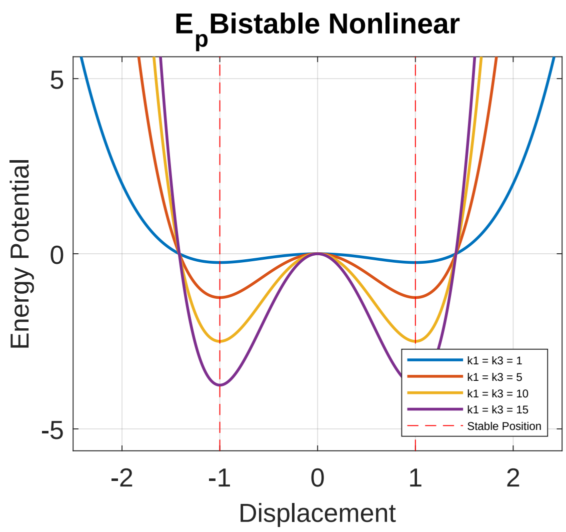

Figure 12: *Potential energy in a bistable system.*

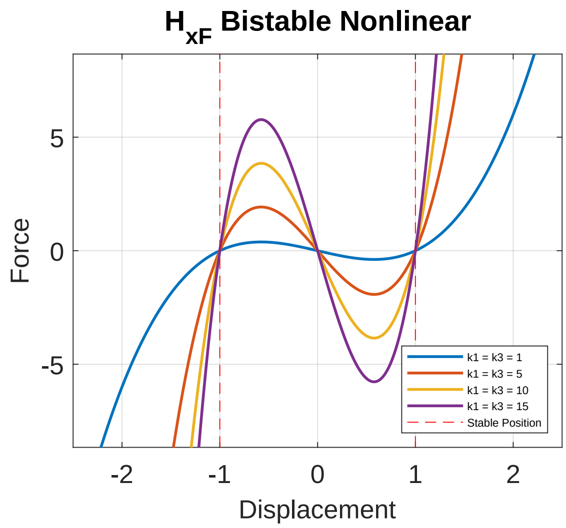

Figure 13: *Force-Displacement curve of the bistable system.*

3.3. Digital Effect Implementation

The digital audio effect is implemented using equations that governs the bistable effect, presented in section 2. The effect output is the velocity $\dot{x}[n]$ calculated using Eq. (7) and Eqs. (10-13). Note, that the input signal of the effect (equivalent the to force), noted as $f[n-1]$ in Eq. (10), is delayed by one sample. Since this delay only adds latency to the effect with no other consequence, it can be replaced by $f[n]$.

The variable parameters that will allow the user to modify the sound effect produced are : 1) input gain control, 2) resonance frequency f_0, 3) damping coefficient, 4) nonlinearity parameter, 5) output gain control. These parameters directly or indirectly modify the variables m, c, k_1, and k_3. To simplify the effect's tuning $k = k_1 = k_3$ is called the nonlinearity parameter. This condition fixes the stable positions of the system respectively at $x = -1$ and $x = 1$. Therefore, by increasing the nonlinearity parameter, the slope of the polynomial curve increases as well as the amount of excitation amplitude required to move across the stable positions.

The resonance frequency parameter allows to adjust the system's resonance frequency, changing the pitch of the sound effect. With the two parameters mentioned previously, the value for the mass is computed from Eq. (2) using only the linear part of the stiffness $k = k_1$ assuming small displacements of the mass to simplify the problem.

A damping parameter is included to control the amount of interwell and intrawell oscillations. Increasing this parameter will reduce sustaining oscillations of the effect and can even restrict the mass from going through a *snapthrough*.

The final algorithm is adjusted for input and output signal levels. The input signal $f[n]$, being a digital signal with values between -1 and 1 is adjustable by the variable input Gain Control. The output signal $\dot{x}[n]$, proportional to velocity is first divided by 10^4 to get the output values near the values corresponding to a digital signal between -1 and 1, and then adjusted by the variable output Gain Control.

3.4. VST Plugin and Online Demo Application

To provide a possibility to play and listen the effect, we built two applications. The fist one is a VST plugin, the second one is an online web-audio application. Both are using the same parameters as the ones described above.

The VST plugin is implemented using the JUCE Framework [25]. The source code, as well as compiled versions for MacOSX and Windows, are available online at `https://ant-novak.com/bistable_effect` [21]. A print-screen of the final VTS plugin is shown in Fig. 14.

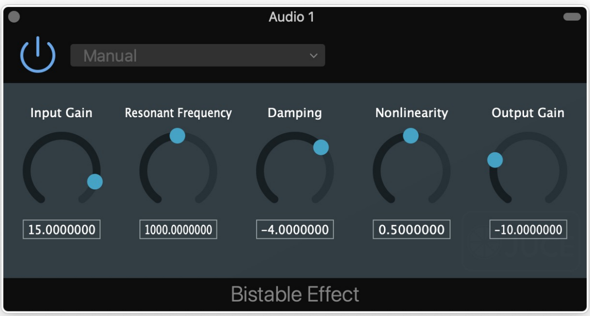

Figure 14: *Bistable Plugin User Guide Interface.*

The online web demo application of the bistable audio effect developed using the Web Audio API [26] is available at `https://ant-novak.com/bistable_effect` [21]. Its print screen is shown in Fig. 15. The input signal is a short loop of a guitar sample. Several presets are available to select different configurations. Another part of the demo application is a graph that shows the shape of the nonlinear bistable curve to visualize in real-time the state of the displacement-force relation. The two grey circles added to this graph show the real-time envelope (maximum and minimum) of the output displacement.

4. CONCLUSION

This work was a part of the master student project in the frame of the International Master's Degree in Electro-Acoustics (IMDEA) program at the Le Mans University (France). Its goal was to implement a bistable system, a system well known from nonlinear dynamics and vibrations research, as a digital audio effect.

Bistability in systems is a complex subject to model and analyze, let alone, to successfully implement its principle into an audio effect. On the other hand, the sounds obtained by this approach are unique and rewarding, leaving a big window for future research and development on the usage of these systems to develop sound effects.

Even though the model is simple to write and implement, achieving a pleasant sound that could be used for sound effects is not an easy task. Indeed, it is crucial to fully understand the operation points of this system to fine-tune its parameters and get the most out of this particular system. For this, the force-displacement curve gives the best insight into how the system performs.

Due to the presence of co-existing steady-state dynamics, the output of a bistable system is different each time, even though the same excitation parameters are used. The bistable effect has no unique sound, but a wide variety of different sounds that depend on the combination of its parameters.

Future work will be focused on application of more sophisticated difference scheme such as energy-consistent numerical methods ([27]) or application of port-Hamiltonian approach [10]), and the accuracy of the discretized bistable system in comparison with the original continuous-time system.

5. ACKNOWLEDGMENTS

This paper is a result of the master student project in the frame of the International Master's Degree in Electro-Acoustics (IMDEA) program at the Le Mans University (France). The students Alexander Ramirez and Vikas Tokala contributed equally to all parts of the project including project management, measurement, coding the VST plugin, building the mechanical prototype, and writing of the paper. Antonin Novak, Frederic Ablitzer, and Manuel Melon only supervised the project and helped with the review process. The web-audio JavaScript applet was written by Antonin Novak based on the C++ code provided by the students.

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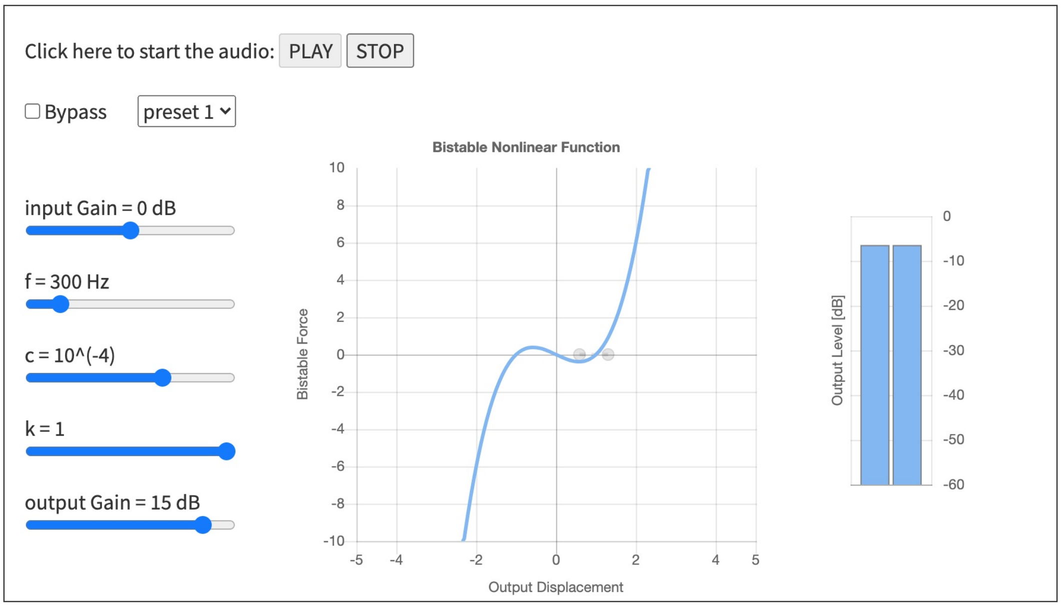

Figure 15: *A screen shot of a bistable effect demo application implemented using Web Audio API, available online at* `https://ant-novak.com/bistable_effect` *[21].*

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A. APPENDIX: C++ CODE

The following C++ code shows the main loop of the bistable effect implementation.

```
for (int sample = 0; sample < buffer.getNumSamples(); sample++)
{
    prepare_parameters();

    // read the new sample (amplify input)
    yL[0] = buffer.getSample(0, sample) * in_gain;
    yR[0] = buffer.getSample(1, sample) * in_gain;

    // displacement equation
    xL[0] = (yL[1] - A2 * xL[1] - A3 * xL[2] - k * pow(xL[1], 3)) / A1;
    xR[0] = (yR[1] - A2 * xR[1] - A3 * xR[2] - k * pow(xR[1], 3)) / A1;

    // velocity (normalized by 10000)
    vL = (xL[0] - xL[1]) / Ts * 0.0001;
    vR = (xR[0] - xR[1]) / Ts * 0.0001;

    // shif buffers
    yL[1] = yL[0]; xL[2] = xL[1]; xL[1] = xL[0];
    yR[1] = yR[0]; xR[2] = xR[1]; xR[1] = xR[0];

    // write output data
    channelData_L[sample] = vL * out_gain ;
    channelData_R[sample] = vR * out_gain ;
}
```

where the function `prepare_parameters()` is defined as

```
void prepare_parameters(){
    // angular frequency
    Wn = 2.0f * MathConstants<float>::pi * Fn;

    m = k / pow(Wn, 2); // mass

    // parameters for dispalcement equation
    A1 = m / pow(Ts, 2) + c / Ts;
    A2 = (-2 * m) / pow(Ts, 2) - c / Ts - k;
    A3 = m / pow(Ts, 2);
}
```